

Stability of Neutrino Mass Degeneracy

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Abstract

Two neutrinos of Majorana masses $m_{1,2}$ with mixing angle θ are unstable against radiative corrections in the limit $m_1 = m_2$, but are stable for $m_1 = -m_2$ (i.e. opposite CP eigenstates) with $\theta = 45^\circ$ which corresponds to an additional symmetry.

Pick two neutrinos, say ν_e and ν_μ . Assume their mass eigenstates to be

$$\nu_1 = \nu_e \cos \theta - \nu_\mu \sin \theta, \quad \nu_2 = \nu_e \sin \theta + \nu_\mu \cos \theta, \quad (1)$$

with eigenvalues m_1 and m_2 respectively. Neutrino oscillations may then occur[1, 2, 3] if both $\Delta m^2 = m_2^2 - m_1^2$ and $\sin^2 2\theta$ are nonzero. However, it is entirely possible that the hierarchy

$$\Delta m^2 \ll m_{1,2}^2 \quad (2)$$

actually exists, so that the smallness of Δm^2 for neutrino oscillations does not necessarily preclude a much larger common mass for the two neutrinos. In fact, this idea is often extended to all three neutrinos[4, 5]. On the other hand, since the charged-lepton masses are all different, radiative corrections[6, 7] to m_1 and m_2 will tend to change Δm^2 as well as θ . This is especially important for the vacuum oscillation solution[8] to the observed solar neutrino deficit[2] which requires $\Delta m^2 \sim 10^{-10} \text{ eV}^2$ and $\sin^2 2\theta \sim 1$. In the following I show that whereas the limit $m_1 = m_2$ is unstable against radiative corrections, the limit $m_1 = -m_2$ and $\theta = 45^\circ$ is stable because it is protected by an additional symmetry. [A negative mass here means that the corresponding Majorana neutrino is odd under CP after a γ_5 rotation to remove the minus sign.]

Consider the 2×2 mass matrix spanning ν_e and ν_μ :

$$\mathcal{M} = \begin{pmatrix} A & B \\ B & C \end{pmatrix}. \quad (3)$$

It has eigenvalues

$$m_{1,2} = \frac{1}{2}(C + A) \mp \frac{1}{2}\sqrt{(C - A)^2 + 4B^2} \quad (4)$$

where

$$A = m_1 \cos^2 \theta + m_2 \sin^2 \theta, \quad (5)$$

$$B = (m_2 - m_1) \sin \theta \cos \theta, \quad (6)$$

$$C = m_1 \sin^2 \theta + m_2 \cos^2 \theta. \quad (7)$$

The mixing angle θ is related to \mathcal{M} according to

$$\tan \theta = \frac{2B}{(C - A) + \sqrt{(C - A)^2 + 4B^2}}, \quad (8)$$

and

$$\Delta m^2 = (C + A)\sqrt{(C - A)^2 + 4B^2}. \quad (9)$$

In the above, I have used the convention $m_2 > |m_1|$ and $0 \leq \theta \leq 45^\circ$.

With radiative corrections, the mass matrix is changed:

$$A \rightarrow A(1 + 2\delta_e), \quad B \rightarrow B(1 + \delta_e + \delta_\mu), \quad C \rightarrow C(1 + 2\delta_\mu). \quad (10)$$

If both e and μ have only gauge interactions, then $\delta_e = \delta_\mu$ and \mathcal{M} is simply renormalized by an overall factor, resulting in

$$\Delta m^2 \rightarrow \Delta m^2(1 + 2\delta)^2, \quad (11)$$

and $\tan \theta$ is unchanged. However, because e and μ have Yukawa interactions proportional to their masses, nontrivial changes do occur in \mathcal{M} . Let

$$\delta = (\delta_\mu + \delta_e)/2, \quad \Delta\delta = \delta_\mu - \delta_e, \quad (12)$$

then

$$\Delta m^2 \rightarrow [(m_2 + m_1)(1 + 2\delta) + (m_2 - m_1)\Delta\delta \cos 2\theta] D, \quad (13)$$

and

$$\tan \theta \rightarrow \frac{(m_2 - m_1) \sin 2\theta(1 + 2\delta)}{(m_2 - m_1) \cos 2\theta(1 + 2\delta) + (m_2 + m_1)\Delta\delta + D}, \quad (14)$$

where

$$D = \sqrt{(m_2 - m_1)^2(1 + 2\delta)^2 + 2\Delta m^2(1 + 2\delta)\Delta\delta \cos 2\theta + (m_2 + m_1)^2(\Delta\delta)^2}. \quad (15)$$

There are two ways for Δm^2 to approach zero:

$$(1) \quad m_2 - m_1 \ll m_2 + m_1 = 2m, \quad (16)$$

and

$$(2) \quad m_2 + m_1 \ll m_2 - m_1 = 2m. \quad (17)$$

In Case (1),

$$D \simeq 2m \sqrt{\left(\frac{\Delta m^2}{4m^2}\right)^2 + 2\left(\frac{\Delta m^2}{4m^2}\right) \Delta\delta \cos 2\theta + (\Delta\delta)^2}. \quad (18)$$

Hence if $\Delta\delta \gg \Delta m^2/4m^2$, then

$$\Delta m^2 \rightarrow 4m^2 \Delta\delta, \quad \tan \theta \rightarrow 0, \quad (19)$$

i.e. this situation is unstable. Of course, if $\Delta m^2/4m^2 \gg \Delta\delta$, there is no problem. For example, if $\Delta m^2 \sim 10^{-3} \text{ eV}^2$ for atmospheric neutrino oscillations[1] and $m \sim 1 \text{ eV}$, then this is easily satisfied. The model-independent contribution to $\Delta\delta$ from the renormalization of the neutrino wavefunctions is

$$\Delta\delta = -\frac{G_F(m_\mu^2 - m_e^2)}{16\pi^2\sqrt{2}} \ln \frac{\Lambda^2}{m_W^2}, \quad (20)$$

where Λ is the scale at which the original mass matrix \mathcal{M} is defined. Other model-dependent contributions[6] to the mass terms themselves may be of the same order. If m_μ is replaced by m_τ in Eq. (20), $\Delta\delta$ is of order 10^{-5} . In that case, only the small-angle matter-enhanced solution[9] to the solar neutrino deficit appears to be stable[7] for $m \sim 1 \text{ eV}$.

In Case (2),

$$D \simeq 2m(1 + 2\delta) \left[1 + \left(\frac{\Delta m^2}{4m^2}\right) \frac{\Delta\delta \cos 2\theta}{(1 + 2\delta)} \right], \quad (21)$$

hence

$$\Delta m^2 \rightarrow \Delta m^2(1 + 2\delta)^2 + 4m^2 \Delta\delta \cos 2\theta(1 + 2\delta), \quad (22)$$

and

$$\tan \theta \rightarrow \tan \theta \left[1 - \left(\frac{\Delta m^2}{4m^2}\right) \Delta\delta \right]. \quad (23)$$

This means that θ is stable and that Δm^2 is also stable if $\cos 2\theta \simeq 0$, i.e. $\theta \simeq 45^\circ$. More precisely, the condition

$$\Delta\delta \cos 2\theta << \frac{\Delta m^2}{4m^2} \quad (24)$$

is required.

Whereas the general form of \mathcal{M} given by Eq. (3) has no special symmetry for the entire theory, the limit $m_1 = -m_2$ and $\theta = 45^\circ$, i.e.

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} \quad (25)$$

is a special case which allows the entire theory to have the additional global symmetry $L_e - L_\mu$. Hence small deviations are protected against radiative corrections, as shown by Eqs.(22) and (23).

The zero $\nu_e - \nu_e$ entry of Eq. (25) also has the well-known virtue of predicting an effective zero ν_e mass in neutrinoless double beta decay. This means that m may be a few eV even though the above experimental upper limit[10] is one order of magnitude less. Hence neutrinos could be candidates for hot dark matter[11] in this scenario.

In conclusion, neutrino mass degeneracy is theoretically viable and phenomenologically desirable provided that $m_1 \simeq -m_2$ and $\theta \simeq 45^\circ$.

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